Algorithm Complexity and Data Structure

AU4606: Network Optimization

AI4702: Network Intelligence and Optimization

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September 18, 2023

Last time

- Basics of graph theory
  - Graphs
  - Paths, cycles, walks
  - Degrees
  - Subgraphs
  - Connectivity
  - Components
  - Acyclic graphs
  - Trees
  - Bipartite graph
- Graph representations
  - Adjacency matrix
  - Incidence matrix
  - Adjacency list
- Network transformations

Today

1. Complexity analysis
   - Complexity measures
   - Asymptotic notation
2. Data structure
   - Why data structure?
   - Stacks and queues
   - d-heaps

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   - Complexity measures
   - Asymptotic notation

2. Data structure
   - Why data structure?
   - Stacks and queues
   - *d*-heaps

Solving a problem

Building blocks for solving a computational problem in computers

- A recipe, or algorithm: a step-by-step procedure
- Means for encoding this procedure in a computational device
- The application of the method to the data of a specific problem

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- Computing resources needed for executing an algorithm
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Time complexity is usually measured in terms of "basic" operations

- Assignment steps
- Arithmetic steps (e.g., addition, subtraction, multiplication, division)
- Logical steps (e.g., conditional statement, comparisons)

# of steps performed by an algorithm = total # of basic operations

Adding two matrices

Algorithm Adding two matrices A and B

for $i = 1 : m$ **do**
  for $j = 1 : n$ **do**
    $C(i,j) = A(i,j) + B(i,j)$
  end for
end for

- $\#$ of additions: mn
- $\#$ of assignments: mn
- Total operations: $2mn$

Perhaps also $\#$ of accessing steps? $2mn$

Complexity measures

- Algorithms are applied to a class of problems
- One algorithm may take different time for different problem instances
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  1. Pros: indicative when solving large number of different instances
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- Average-case analysis: analyze alg. on instances and take average
  1. Pros: indicative when solving large number of different instances
  2. Cons: distributions of problem instances; difficult analysis
- Worst-case analysis: analyze algorithm on "hardest" instance
  1. Pros: provides conclusive guarantees on how algorithms perform
  2. Cons: pathological cases

How to measure complexity

Algorithm Adding two matrices A and B

```
for i = 1 : m do
    for j = 1 : n do
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```

Takes roughly $2mn$ basic operations (time steps)

- Number of basic steps required depends on the problem instance
- Measure the complexity of algorithms in terms of "problem sizes"

Problem sizes

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for i = 1 : m do
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```

Problem sizes: # of bits to encode the problem data

- Adding matrices: $mn \log_2 M$, where M largest element in A and B
- Network flow problem
  1. Number of nodes n
  2. Number of arcs m
  3. Arc cost coefficient c_{ij}
  4. Arc capacity u_{ij}

  problem size approximately:

$$n \log n + m \log m + m \log C + m \log U$$

where $C = \max_{(i,j)\in A} c_{ij}$ and $U = \max_{(i,j)\in A} u_{ij}$

Polynomial time algorithms

- Polynomial-time algorithm: worst-case complexity is bounded by a polynomial function of the problem size, i.e., it is a polynomial function of n, m, $\log C$, and $\log U$
  - mn
  - n^2
  - $m + n \log C$
- Strongly polynomial-time algorithm if does not involve $\log C$ or $\log U$,
  - n
  - $n^2 m$
- Otherwise, a weakly polynomial-time algorithm
  - $m + n \log C$

Note: algorithms having complexity mnU is exponential!

Algorithm complexity with asymptotic notations

- We usually only care about the order of # of steps
- Ignore (distracting) constant factors

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Usually written as $O(mn)$

Asymptotic notation: big oh

Definition of Big Oh

Given two nonnegative functions $f, g : \mathbb{R} \to \mathbb{R}$, we say that

$$f = O(g)$$

if

$$\lim_{x \to \infty} \frac{f(x)}{g(x)} < \infty$$

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For $f, g : \mathbb{R} \to \mathbb{R}$, we say that $f = O(g)$ if there exists a constant $c > 0$ and an x_0 such that for all $x \geq x_0$, $f(x) \leq cg(x)$.

- $2x = O(x)$
- $x = O(x^2)$
- $10^8 x^2 + 3x + 2 = O(x^2)$
- $2^x + x^{10000} + 3 = O(2^x)$
- $c = O(1)$ for any $c > 0$

Suppose you want to make a statement of the form "the running time of the algorithm is a least...". Can you say it is "at least $O(n^2)$"?

Asymptotic notation: big omega

Suppose you want to make a statement of the form "the running time of the algorithm is a least...". Can you say it is "at least $O(n^2)$"? NO!

Definition of Big Omega

Given two nonnegative functions $f, g : \mathbb{R} \to \mathbb{R}$, we say that

$$f = \Omega(g)$$

if there exists a constant $c > 0$ and an x_0 such that for all $x \geq x_0$, $f(x) \geq cg(x)$.

Examples

- $x^2 = \Omega(x)$
- $2^x = \Omega(x^2)$
- $\frac{x}{100} = \Omega(100x + 25)$

Big Oh and Big Omega

$f(x) = O(g(x))$ if and only if $g(x) = \Omega(f(x))$.

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What if we want to say some function is "strictly dominated" by another?

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Examples

- $x^{0.99999} = o(x)$
- $\log x = o(x^{\epsilon})$ for any $\epsilon > 0$
- $\frac{1}{x} = o(1)$

Asymptotic notation: little oh

Maximum Flow and Minimum-Cost Flow in Almost-Linear Time

Publisher: IEEE Cite This PDF

Li Chen ; Rasmus Kyng ; Yang P. Liu ; Richard Peng ; Maximilian Probst Gutenberg ; Sushant Sachdeva **All Authors**

Abstract:
We give an algorithm that computes exact maximum flows and minimum-cost flows on directed graphs with m edges and polynomially bounded integral demands, costs, and capacities in $m^{1+o(1)}$ time. Our algorithm builds the flow through a sequence of $m^{1+o(1)}$ approximate undirected minimum-ratio cycles, each of which is computed and processed in amortized $m^{o(1)}$ time using a new dynamic graph data structure. Our framework extends to algorithms running in $m^{1+o(1)}$ time for computing flows that minimize general edge-separable convex functions to high accuracy. This gives almost-linear time algorithms for several problems including entropy-regularized optimal transport, matrix scaling, p-norm flows, and p-norm isotonic regression on arbitrary directed acyclic graphs.

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$$f = \omega(g)$$

if

$$\lim_{x \to \infty} \frac{g(x)}{f(x)} = 0$$

Little Oh and Little Omega

$f(x) = o(g(x))$ if and only if $g(x) = \omega(f(x))$.

Examples

- $x^{1.5} = \omega(x)$
- $\sqrt{x} = \omega(\log^2 x)$

Asymptotic notation: theta

Definition of Theta

Given two nonnegative functions $f, g : \mathbb{R} \to \mathbb{R}$, then

$$f = \Theta(g) \qquad \text{if and only if} \qquad f = O(g) \text{ and } g = O(f)$$

Two functions grow equally fast

Examples

- $10x^3 - 20x^2 + 1 = \Theta(x^3)$
- $\pi^2 3^{x-7} + \frac{(2.7x^{133}+x^9-86)^4}{\sqrt{x}} - 1.08^{3x} = \Theta(3^x)$

Asymptotic notation: tilde

Definition of Tilde

Given two nonnegative functions $f, g : \mathbb{R} \to \mathbb{R}$, we say f is asymptotically equal to g, in symbols,

$$f \sim g$$

if

$$\lim_{x\to\infty} \frac{g(x)}{f(x)} = 1$$

Immediately

$$f \sim g \implies \begin{cases} f = O(g), \\ g = O(f), \\ f = \Theta(g). \end{cases}$$

- $\frac{1}{2}x^2 + 3x - 2 \sim \frac{1}{2}x^2$
- $e^x + 3x^2 \sim e^x$

Asymptotic notation: confusions

We know that

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More mathematically precise notation is

$$f \in O(g)$$

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More mathematically precise notation is

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In fact, people write

- $f = O(g)$
- $f \leq O(g)$
- f is $O(g)$
- $f \in O(g)$

to mean the same thing

Asymptotic notation: intuitions

O	"means"	$\leq$
Ω	"means"	$\geq$
o	"means"	$<$
ω	"means"	$>$
Θ	"means"	$=$

Asymptotic notation: exercises

$$O \qquad \Omega \qquad o \qquad \omega \qquad \Theta$$

$$2n + \log n = \boxed{}(n)$$

$$\log n = \boxed{}(n)$$

$$\sqrt{n} = \boxed{}(\log^{300} n)$$

$$n2^n = \boxed{}(n)$$

$$n^2 = \boxed{}(1.01^n)$$

Mental break: galactic algorithm

Galactic algorithm (hiding constant factors)

- A galactic algorithm is one that outperforms other algorithms for problems that are sufficiently large, but where "sufficiently large" is so big that the algorithm is never used in practice

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- Matrix multiplication
  - Naive algorithm takes $O(n^3)$
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Problem complexity vs algorithm complexity

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Problem complexity vs algorithm complexity

- Problem complexity: how much time does best algorithm take to solve
- Algorithm complexity: how much time does algorithm solve worst case

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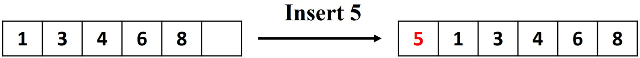
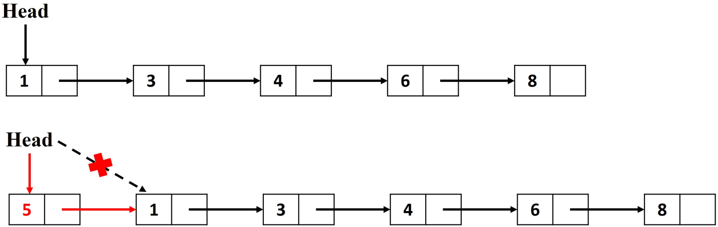
   - Complexity measures
   - Asymptotic notation
2. Data structure
   - Why data structure?
   - Stacks and queues
   - *d*-heaps

Why data structure?

- Operations can take different time on different data structure



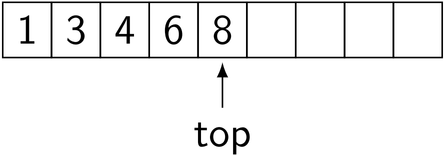
$O(n)$ time



$O(1)$ time

Two important data structures: stacks

- A stack is a special kind of ordered list (or set) in which all insertions and deletions take place at one end, called the top
  - Last-in-first-out



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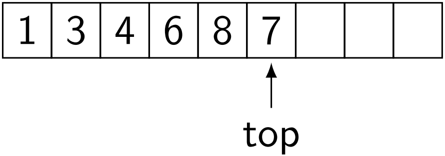

Add 7 to the stack

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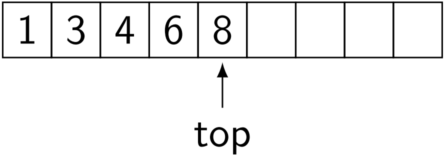

Remove 7 from the stack

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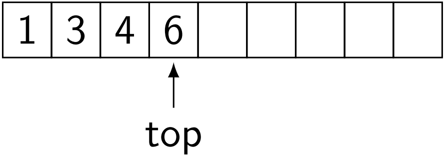

Remove 8 from the stack

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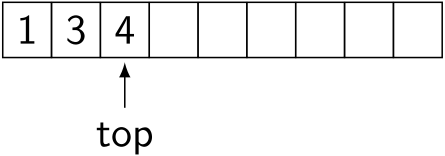

Remove 6 from the stack

Two important data structures: queues

- A queue is another special kind of list, with elements inserted at one end (the rear) and deleted from the other end (the front)
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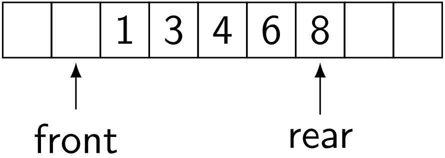

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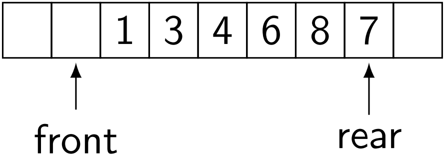

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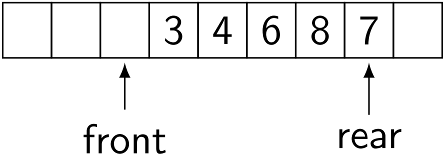

1 leaves the queue

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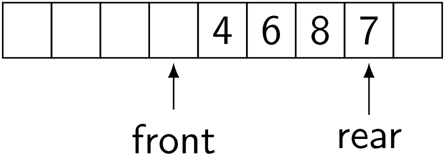
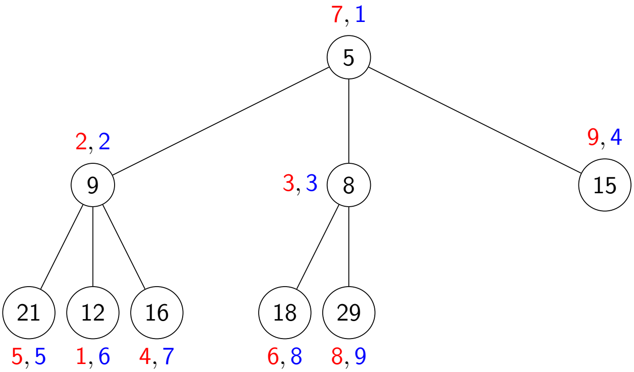


3 leaves the queue

d-heaps: operations

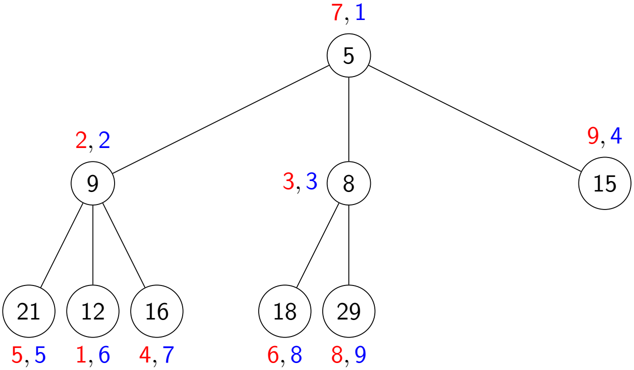
- Store and manipulate a collection H of elements when each element $i \in H$ has an associated real number $key(i)$
  - In shortest path problems, H is graph nodes, $key(i)$ is path length
- Basic operations
  1. create(H): create an empty heap H
  2. insert(i, H): insert an element i in the heap.
  3. find-min(i, H): find an element i with the minimum key in the heap.
  4. delete-min(i, H): delete the element i with the minimum key
  5. delete(i, H): delete an arbitrary element i from the heap.
  6. decrease-key(i, $value$, H): decrease the $key(i)$ to a smaller $value$
  7. increase-key(i, $value$, H): increase the $key(i)$ to a larger $value$
- The elements are stored as a rooted tree

d-heaps: properties

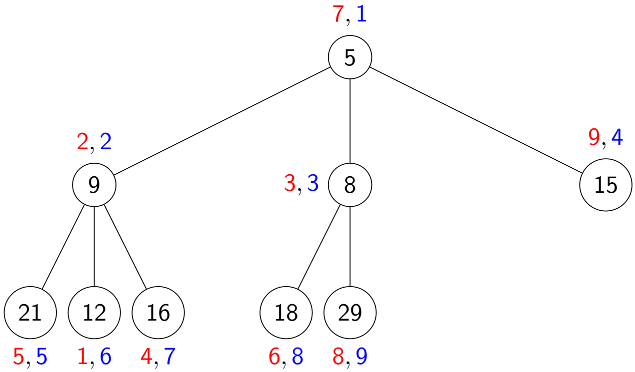


- Keys of elements are shown in the rooted tree
  1. Red indices are indices of elements (e.g., graph nodes)
  2. Blue indices are indices of elements in the tree
- Each node has at most d successors

d-heaps: properties

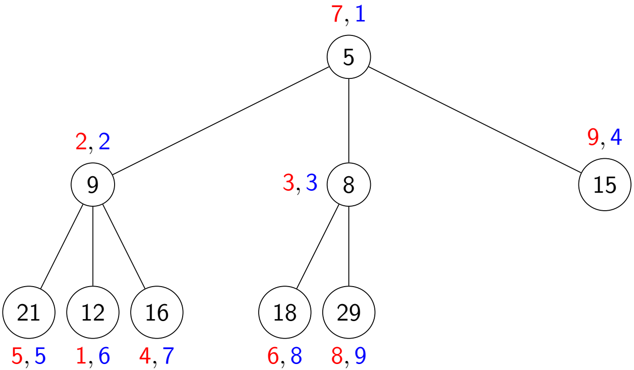


- Depth of a node: the number of arcs in the unique path to the root
  - node 8 has depth 2
- Nodes added in increasing order of depth values, and for the same depth, from left to right
  1. At most d^k nodes in depth k
  2. At most $(d^{k+1} - 1)/(d - 1)$ nodes between depth 0 and k
  3. The depth of an n-node d-heap is at most $\lfloor \log_d n \rfloor$



- Using an array with *last* being the number of nodes
  DHEAP=
  $\left[7:5,\ 2:9,\ 3:8,\ 9:15,\ 5:21,\ 1:12,\ 4:16,\ 6:18,\ 8:29\right]$
  last $= 9$
- Position array: position(i) $= j$, e.g., position(3) $= 3$, position(6) $= 8$

d-heaps: accessing predecessors and successors



- Predecessor of node in position i is in position $\lceil (i-1)/d \rceil$
  e.g., Pred(8)=$\lceil (8-1)/3 \rceil$=3; Pred(6)=$\lceil (6-1)/3 \rceil$=2
- Successors of node in position i are in positions $id - d + 2, \ldots, id + 1$
  e.g., Succ(2)=$\{5, 6, 7\}$

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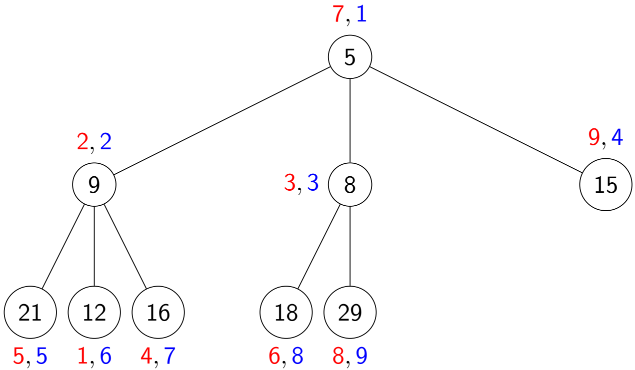

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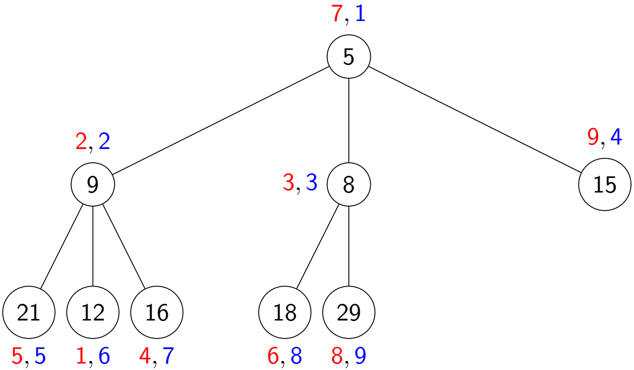
  e.g., Pred(6)= 3; Pred(1)=2
- Successors of node in position i are in positions $id - d + 2, \ldots, id + 1$
  e.g., Succ(2)={1, 4, 5}

d-heaps: order property



- Key of node i is less than or equal to each of its successors, i.e., $key(i) \leq key(j)$ for $j \in \text{Succ}(i)$
- The root node of the d-heap has the smallest key

d-heaps: swapping

- Heap operations are reduced to swaps that take $O(1)$ time
- swap(i, j): swap the positions of i and j

  before swap$(2, 7)$:

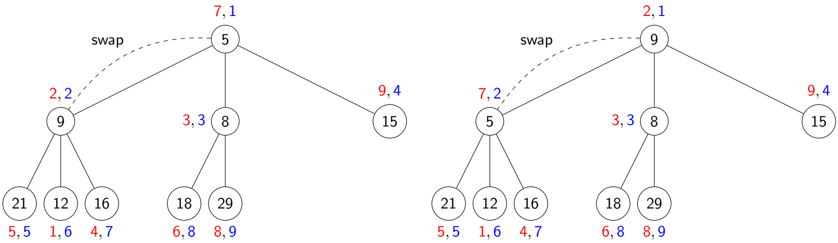
  $[7 : 5,\ 2 : 9,\ 3 : 8,\ 9 : 15,\ 5 : 21,\ 1 : 12,\ 4 : 16,\ 6 : 18,\ 8 : 29]$

  position$(2) = 2$, position$(7) = 1$

  after swap$(2, 7)$:

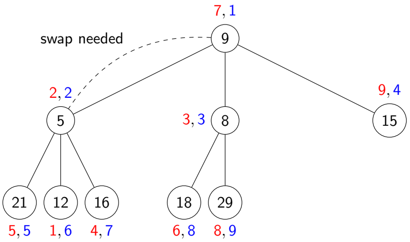
  $[2 : 9,\ 7 : 5,\ 3 : 8,\ 9 : 15,\ 5 : 21,\ 1 : 12,\ 4 : 16,\ 6 : 18,\ 8 : 29]$

  position$(2) = 1$, position$(7) = 2$

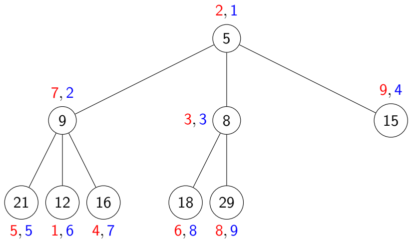


(a) Before swap (b) After swap

- Recall order property: $key(i) \leq key(j)$ for $j \in \text{Succ}(i)$
- Suppose $key(j)$ decreases and $key(j) < key(i)$ for some $j \in \text{Succ}(i)$
  - sift up



(a) Before swap

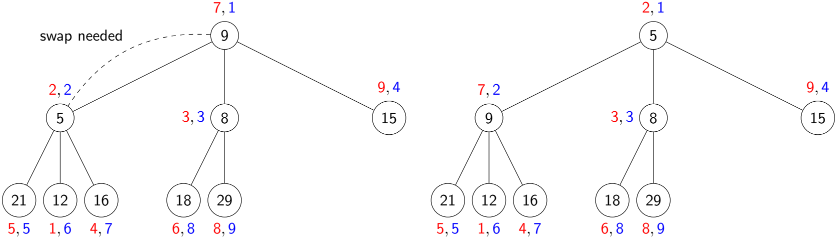


(b) After swap

$key(2)$ decreases to 5

- If node's key decreases, takes at most $O(\log_d n)$ to restore order

- Recall order property: $key(i) \leq key(j)$ for $j \in \text{Succ}(i)$
- Suppose $key(i)$ increases and $key(i) > key(j)$ for some $j \in \text{Succ}(i)$
  - sift down



(a) Before swap (b) After swap

$key(7)$ increases to 9

- If node's key increases, takes at most $O(d \cdot log_d n)$ to restore order

d-heaps: performing heap operations

1. find-min(i, H): root node, $O(1)$
2. insert(i, H): inset to the end, and swap up, $O(\log_d n)$
3. decrease-key(i, *value*, H): swap up $O(\log_d n)$
4. delete-min(i, H): make last node root, swap down $O(d \cdot log_d n)$
5. delete(i, H): fill with last node, swap down $O(d \cdot log_d n)$
6. increase-key(i, *value*, H): swap down $O(d \cdot log_d n)$

Sorting n elements?

d-heaps: performing heap operations

1. find-min(i, H): root node, $O(1)$
2. insert(i, H): inset to the end, and swap up, $O(\log_d n)$
3. decrease-key(i, $value$, H): swap up $O(\log_d n)$
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6. increase-key(i, $value$, H): swap down $O(d \cdot log_d n)$

Sorting n elements?

1. Create a d-heap: add one at a time and swap up $O(n \log_d n)$

d-heaps: performing heap operations

1. find-min(i, H): root node, $O(1)$
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Sorting n elements?

1. Create a d-heap: add one at a time and swap up $O(n \log_d n)$
2. Find minimum element and delete it n times, $O(n) + O(nd \cdot \log_d n)$

d-heaps: performing heap operations

1. find-min(i, H): root node, $O(1)$
2. insert(i, H): inset to the end, and swap up, $O(\log_d n)$
3. decrease-key(i, $value$, H): swap up $O(\log_d n)$
4. delete-min(i, H): make last node root, swap down $O(d \cdot log_d n)$
5. delete(i, H): fill with last node, swap down $O(d \cdot log_d n)$
6. increase-key(i, $value$, H): swap down $O(d \cdot log_d n)$

Sorting n elements?

1. Create a d-heap: add one at a time and swap up $O(n \log_d n)$
2. Find minimum element and delete it n times, $O(n) + O(nd \cdot \log_d n)$
3. Total: $O(nd \cdot \log_d n)$

Upcoming

Week 1-8 (AU4606 & AI4702):

- Introduction (1 lecture)
- Preparations (3 lectures)
  - basics of graph theory
  - algorithm complexity and data structure (this lecture)
  - graph search algorithm (next lecture)
- Shortest path problems (3 lectures)
- Maximum flow problems (5 lectures)
- Minimum cost flow problems (3 lectures)
- Introduction to multi-agent systems (1 lecture)
- Introduction to cloud networks (1 lecture)

Week 9-16 (AU4606):

- Simplex and network simplex methods (2 lectures)
- Global minimum cut problems (3 lectures)
- Minimum spanning tree problems (3 lectures)