Motivating Problems and Systems

AU7036: Introduction to Multi-agent Systems

Xiaoming Duan
Department of Automation
Shanghai Jiao Tong University

February 20, 2024

Welcome

- Welcome to AU7036: Introduction to Multi-agent Systems

- Instructors
  - Xiaoming Duan (26): averaging systems and flow dynamics
  - Peng Wang (22): nonlinear (time-varying) systems and advanced topics

- Main references
  - Francesco Bullo, *Lectures on Network Systems*, Kindle Direct Publishing, version 1.6, Sep 1, 2022. `https://fbullo.github.io/lns/`. Chinese translation is available at `https://jbox.sjtu.edu.cn/l/B1pjsb` (Ongoing project)
  - Wei Ren, Yongcan Cao. *Distributed Coordination of Multi-agent Networks: Emergent Problems, Models, and Issues*, Springer London, 2011.
  - Wei Ren, Randal W. Beard. *Distributed Consensus in Multi-vehicle Cooperative Control: Theory and Applications*, Springer London, 2008.
  - Fei Chen, Wei Ren. *Distributed Average Tracking in Multi-agent Systems*, Springer Cham, 2020.

Course content

Week 1-6:

- Introduction
- Elements of matrix theory
- Elements of graph theory
- Elements of algebraic graph theory
- Discrete-time averaging systems
- The Laplacian matrix
- Continuous-time averaging systems
- Diffusively-coupled linear systems
- (*) The incidence matrix and its applications
- (*) Metzler matrices and dynamical flow systems

Week 7-14:

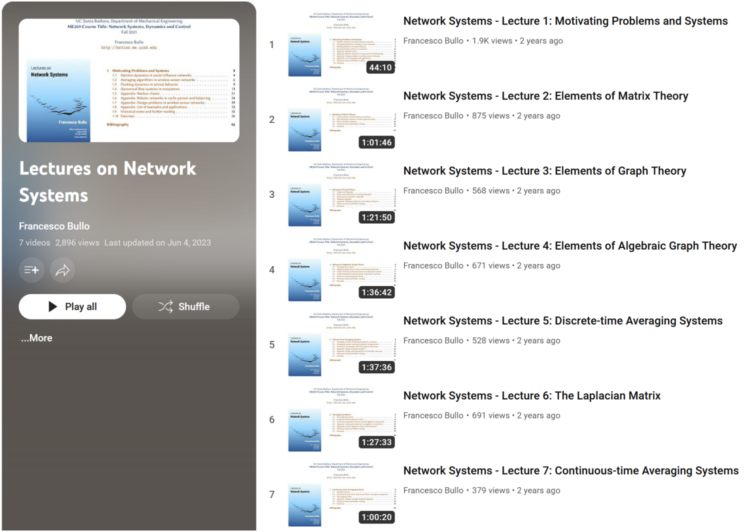
- Lyapunov stability theory
- Nonlienar averaging systems (Euler-Lagrangian, oscillators)
- Other advanced topics

Week 15-16:

- Project presentation

Course content

Week 1-6:



UC Santa Barbara, Department of Mechanical Engineering
ME269 Course Title: Network Systems, Dynamics and Control
Fall 2021

Francesco Bullo
http://motion.me.ucsb.edu

Lectures on
Network Systems
Francesco Bullo

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Lectures on Network Systems

Francesco Bullo

7 videos 2,896 views Last updated on Jun 4, 2023

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Francesco Bullo • 1.9K views • 2 years ago

2 **Network Systems - Lecture 2: Elements of Matrix Theory** 1:01:46
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3 **Network Systems - Lecture 3: Elements of Graph Theory** 1:21:50
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4 **Network Systems - Lecture 4: Elements of Algebraic Graph Theory** 1:36:42
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5 **Network Systems - Lecture 5: Discrete-time Averaging Systems** 1:37:36
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6 **Network Systems - Lecture 6: The Laplacian Matrix** 1:27:33
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7 **Network Systems - Lecture 7: Continuous-time Averaging Systems** 1:00:20
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Class times and grading

Class times

- Present theory
- Do proofs (mostly involve system theory, matrix and graph theory)

Grading

- 40% homework: 6 problem sets (might involve coding)
- 60% course project
    - A presentation on related topics (30%)
    - A final report (30%)

Disclaimer

- We do not study anything related to (artificial) intelligence

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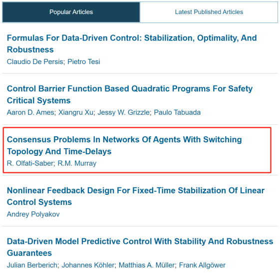

Formulas For Data-Driven Control: Stabilization, Optimality, And Robustness
Claudio De Persis; Pietro Tesi

Control Barrier Function Based Quadratic Programs For Safety Critical Systems
Aaron D. Ames; Xiangru Xu; Jessy W. Grizzle; Paulo Tabuada

Consensus Problems In Networks Of Agents With Switching Topology And Time-Delays
R. Olfati-Saber; R.M. Murray

Nonlinear Feedback Design For Fixed-Time Stabilization Of Linear Control Systems
Andrey Polyakov

Data-Driven Model Predictive Control With Stability And Robustness Guarantees
Julian Berberich; Johannes Köhler; Matthias A. Müller; Frank Allgöwer

Consensus problems in **networks** of **agents** with **switching topology** and **time-delays**
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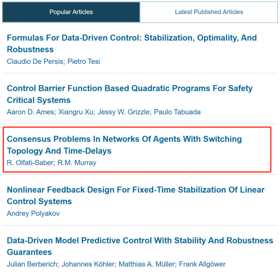

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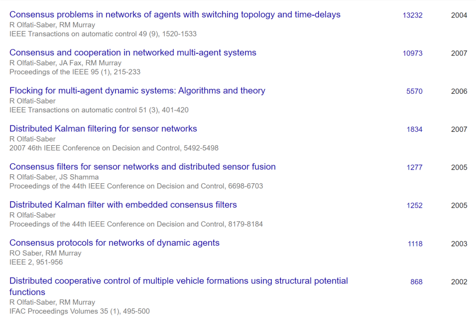


Consensus problems in networks of agents with switching topology and time-delays R Olfati-Saber, RM Murray IEEE Transactions on automatic control 49 (9), 1520-1533	13232	2004
Consensus and cooperation in networked multi-agent systems R Olfati-Saber, JA Fax, RM Murray Proceedings of the IEEE 95 (1), 215-233	10973	2007
Flocking for multi-agent dynamic systems: Algorithms and theory R Olfati-Saber IEEE Transactions on automatic control 51 (3), 401-420	5570	2006
Distributed Kalman filtering for sensor networks R Olfati-Saber 2007 46th IEEE Conference on Decision and Control, 5492-5498	1834	2007
Consensus filters for sensor networks and distributed sensor fusion R Olfati-Saber, JS Shamma Proceedings of the 44th IEEE Conference on Decision and Control, 6698-6703	1277	2005
Distributed Kalman filter with embedded consensus filters R Olfati-Saber Proceedings of the 44th IEEE Conference on Decision and Control, 8179-8184	1252	2005
Consensus protocols for networks of dynamic agents RO Saber, RM Murray IEEE 2, 951-956	1118	2003
Distributed cooperative control of multiple vehicle formations using structural potential functions R Olfati-Saber, RM Murray IFAC Proceedings Volumes 35 (1), 495-500	868	2002

Questions?

Today

Motivating problems

- Opinion dynamics
- Averaging in wireless sensor networks
- Flocking dynamics
- Distributed parameter estimation

Synchronization in nature

Video available at `https://www.ted.com/talks/steven_strogatz_the_science_of_sync`

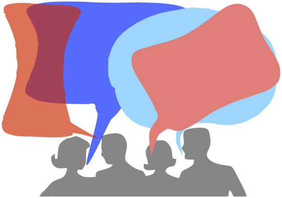
Steven Strogatz: The science of sync, 2008

Coordination of engineered systems

Video available at `https://www.ted.com/talks/vijay_kumar_robots_that_fly_and_cooperate?language=en`

Vijay Kumar: Robots that fly ... and cooperate, 2012

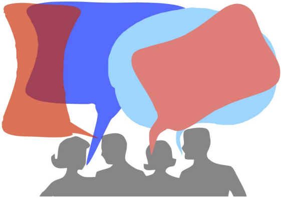
Motivating example: opinion dynamics



Interactions in a social influence network

- A group of n individuals who must act together as a team

Motivating example: opinion dynamics



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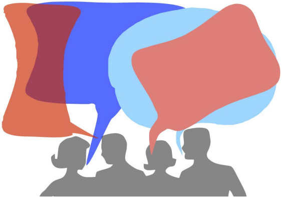

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- Individual i is influenced by other members $j \neq i$ of the group

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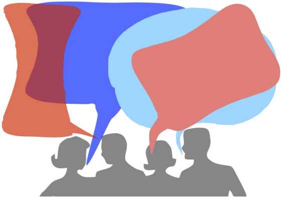
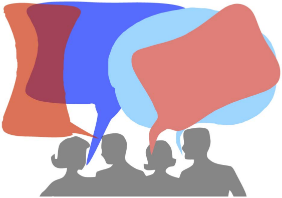


Interactions in a social influence network

- A group of n individuals who must act together as a team
- Each individual i has its own opinion p_i for some estimate/event
- Individual i is influenced by other members $j \neq i$ of the group
- How to model predictions that the individual will revise its estimate?

Motivating example: opinion dynamics

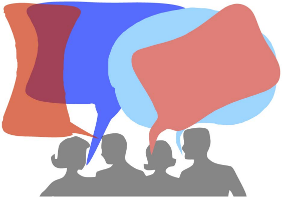


Interactions in a social influence network

The French-Harary-DeGroot model predicts that

$$p_i(k+1) = \sum_{j=1}^{n} a_{ij} p_j(k)$$

Motivating example: opinion dynamics



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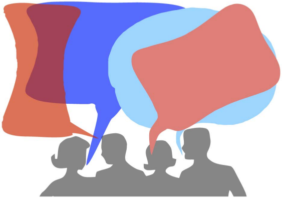
$$p_i(k+1) = \sum_{j=1}^{n} a_{ij} p_j(k)$$

- $a_{ij} \geq 0$ denotes the weight that individual i assigns to individual j
  - a_{ii} describes the attachment of individual i to its own opinion
  - a_{ij} is an interpersonal influence weight that i accords to j
- $\sum_{j=1}^{n} a_{ij} = 1$ for all i

Motivating example: opinion dynamics



Interactions in a social influence network

The French-Harary-DeGroot model predicts that

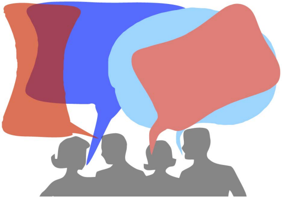
$$p_i(k+1) = \sum_{j=1}^{n} a_{ij} p_j(k)$$

In matrix form

$$p(k+1) = Ap(k)$$

where $A = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}$

Motivating example: opinion dynamics



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- A is entry-wise nonnegative
- Each row has unit sum

Motivating example: opinion dynamics

The French-Harary-DeGroot model

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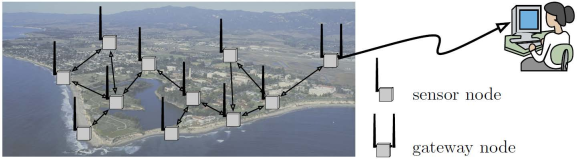
Scientific questions of interest

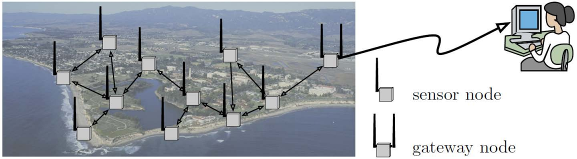
- Is this model of opinion dynamics believable (empirical evidence)?
- How does one measure the coefficients a_{ij}?
- Are there more realistic, empirically-motivated models?
- Conditions for convergence? What is the final estimate?

Motivating example: wireless sensor networks



A wireless sensor network

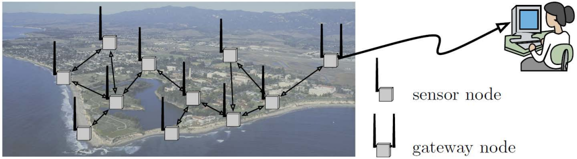
Motivating example: wireless sensor networks



A wireless sensor network

- A collection of spatially-distributed sensing/computing devices
    - Measure environmental variables (e.g., temperature, sound, light, etc.)
    - Perform local computations and transmit information to neighbors

Motivating example: wireless sensor networks



A wireless sensor network

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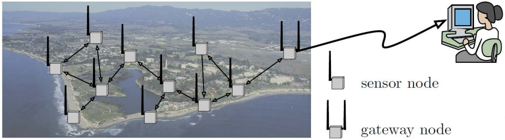
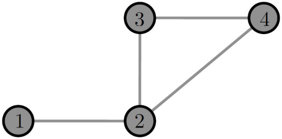
  - Measure environmental variables (e.g., temperature, sound, light, etc.)
  - Perform local computations and transmit information to neighbors

How can all devices obtain an accurate estimate in a distributed way?

Motivating example: wireless sensor networks

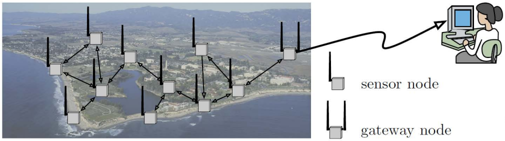


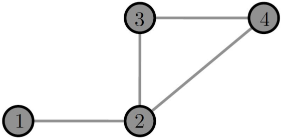


Wireless sensor networks: linear averaging

- Each node has a measured temperature $x_i(0)$

Motivating example: wireless sensor networks





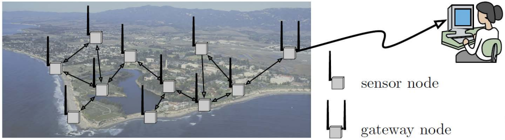
Wireless sensor networks: linear averaging

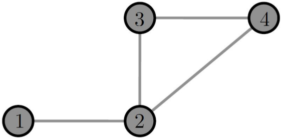
- Each node has a measured temperature $x_i(0)$
- Apply the following linear averaging algorithm

$$x_i(k+1) = \text{ average}(x_i(k), \{x_j(k), \text{for all neighbors } j\})$$

  - e.g., $x_1(k+1) = x_1(k)/2 + x_2(k)/2$

Motivating example: wireless sensor networks





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  - e.g., $x_1(k+1) = x_1(k)/2 + x_2(k)/2$

- Update rule $x(k+1) = Ax(k)$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}$$

Again we have a nonnegative, unit-row-sum matrix

Motivating example: wireless sensor networks

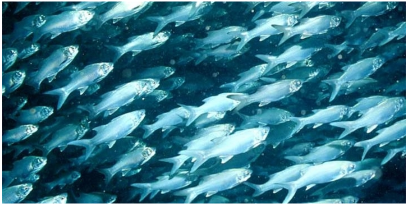
Wireless sensor networks: linear averaging

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 & 0 \\ 1/4 & 1/4 & 1/4 & 1/4 \\ 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1/3 & 1/3 & 1/3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix}$$

Scientific questions of interest

- Does the algorithm converge? Do all nodes agree?
- Is the final value equal to the average of the initial conditions?
- Conditions on the graph and the matrix for the algorithm to converge?
- How quick is the convergence?

Motivating example: flocking dynamics



Flocking dynamics: a simple alignment rule

- Each animal steers towards the average heading of its neighbors

$$\dot{\theta}_i = \begin{cases} (\theta_j - \theta_i), & \text{if one neighbor} \\ \frac{1}{2}(\theta_{j_1} - \theta_i) + \frac{1}{2}(\theta_{j_2} - \theta_i), & \text{if two neighbors} \\ \frac{1}{m}(\theta_{j_1} - \theta_i) + \cdots + \frac{1}{m}(\theta_{j_m} - \theta_i), & \text{if } m \text{ neighbors} \end{cases}$$

$$= \text{average}\big(\{\theta_j, \text{ for all neighbors } j\}\big) - \theta_i$$

Motivating example: flocking dynamics

Flocking dynamics: a simple alignment rule

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$$= \text{average}\big(\{\theta_j, \text{ for all neighbors } j\}\big) - \theta_i$$

In matrix form

$$\dot{\theta} = A\theta - \theta = (A - I)\theta$$

Motivating example: flocking dynamics

Flocking dynamics: a simple alignment rule

$$\begin{aligned}\dot{\theta}_i &= \begin{cases}(\theta_j - \theta_i), & \text{if one neighbor}\\ \frac{1}{2}(\theta_{j_1} - \theta_i) + \frac{1}{2}(\theta_{j_2} - \theta_i), & \text{if two neighbors}\\ \frac{1}{m}(\theta_{j_1} - \theta_i) + \cdots + \frac{1}{m}(\theta_{j_m} - \theta_i), & \text{if } m \text{ neighbors}\end{cases}\\ &= \text{average}\big(\{\theta_j,\ \text{for all neighbors } j\}\big) - \theta_i\end{aligned}$$

In matrix form

$$\dot{\theta} = A\theta - \theta = (A - I)\theta$$

Scientific questions of interest

- How valid is the model in understanding/reproducing the behavior?
- What are equilibrium headings and when are they attractive?
- Conditions for the graph to ensure a proper flocking behavior?

Motivating example: distributed parameter estimation

Distributed parameter estimation

$$y_i = B_i\theta + v_i, \text{ for all } i \in \{1, \cdots, n\}$$

where

- $\theta \in \mathbb{R}^m$ is an unknown parameter to be estimated via measurements
- $y_i \in \mathbb{R}^{m_i}$ is the measurement
- B_i is a known measurement matrix
- v_i is random measurement noise

Motivating example: distributed parameter estimation

Distributed parameter estimation

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- B_i is a known measurement matrix
- v_i is random measurement noise

Assumption

- Vectors $v_1, \cdots, v_n$ are independent jointly Gaussian with
    - Zero-mean $\mathbb{E}[v_i] = 0_{m_i}$ and
    - Positive-definite covariance $\mathbb{E}[v_i v_i^\top] = \Sigma_i = \Sigma_i^\top$
- Measurement parameters satisfy: $\sum_i m_i \geq m$ and $\begin{bmatrix} B_1 \\ \vdots \\ B_n \end{bmatrix}$ is full rank.

Motivating example: distributed parameter estimation

Distributed parameter estimation

$$y_i = B_i\theta + v_i, \text{ for all } i \in \{1, \cdots, n\}$$

We aim to minimize the following weighted least-square error

$$\min_{\theta} \sum_{i=1}^{n} \left(y_i - B_i\theta\right)^{\top} \Sigma_i^{-1} \left(y_i - B_i\theta\right)$$

Motivating example: distributed parameter estimation

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$$\min_{\theta} \sum_{i=1}^{n} \big(y_i - B_i\theta\big)^{\top} \Sigma_i^{-1} \big(y_i - B_i\theta\big)$$

The solution is given by

$$\theta^* = \Big(\sum_{i=1}^{n} B_i^{\top} \Sigma_i^{-1} B_i\Big)^{-1} \sum_{i=1}^{n} B_i^{\top} \Sigma_i^{-1} y_i$$

How do we compute θ^* in a distributed way?

Motivating example: distributed parameter estimation

$$\theta^* = \Big(\sum_{i=1}^{n} B_i^\top \Sigma_i^{-1} B_i \Big)^{-1} \sum_{i=1}^{n} B_i^\top \Sigma_i^{-1} y_i$$

1. Compute the average of the quantities $B_i^\top \Sigma_i^{-1} B_i$ and $B_i^\top \Sigma_i^{-1} y_i$

Motivating example: distributed parameter estimation

$$\theta^* = \Big(\sum_{i=1}^{n} B_i^\top \Sigma_i^{-1} B_i\Big)^{-1} \sum_{i=1}^{n} B_i^\top \Sigma_i^{-1} y_i$$

1. Compute the average of the quantities $B_i^\top \Sigma_i^{-1} B_i$ and $B_i^\top \Sigma_i^{-1} y_i$
2. Compute the optimal estimate via

$$\begin{aligned}\hat{\theta}^* &= \mathrm{average}(B_1^\top \Sigma_1^{-1} B_1, \cdots, B_n^\top \Sigma_n^{-1} B_n)^{-1} \\ &\quad \times \mathrm{average}(B_1^\top \Sigma_1^{-1} y_1, \cdots, B_n^\top \Sigma_n^{-1} y_n)\end{aligned}$$

Upcoming

Week 1-6:

- Introduction
- Elements of matrix theory
- Elements of graph theory
- Elements of algebraic graph theory
- Discrete-time averaging systems
- The Laplacian matrix
- Continuous-time averaging systems
- Diffusively-coupled linear systems
- (*) The incidence matrix and its applications
- (*) Metzler matrices and dynamical flow systems

Week 7-14:

- Lyapunov stability theory
- Nonlienar averaging systems (Euler-Lagrangian, oscillators)
- Other advanced topics

Week 15-16:

- Project presentation