## Erratum

- There is a fixable error in the proof of Lemma 8 where we falsely claimed $\left(1-\frac{1}{n}\right)^{\tau} \leq \frac{1}{2}$ (a counterexample would be the case when $n=4$ and $\tau=2$ ). We hereby provide another proof to show that $\frac{d g(\tau)}{\tau} \leq 0$.
To show that

$$
\left(1+\frac{\tau}{n-1}\right)\left(1-\frac{1}{n}\right)^{\tau}-1 \leq 0
$$

we let $p(\tau)=(1+\tau /(n-1))(1-1 / n)^{\tau}$. Then, we need to prove $p(\tau) \leq 1$ for all $1 \leq \tau \leq n-1$. Consider $q(\tau)=\log p(\tau)$, then we need to prove $q(\tau) \leq 0$ for all $1 \leq \tau \leq n-1$. Note that $q(1)=0$ $(p(1)=1)$. It is easy to derive

$$
\frac{d q}{d \tau}=\frac{1}{n+\tau-1}+\log \left(1-\frac{1}{n}\right) \leq \frac{1}{n+\tau-1}-\frac{1}{n} \leq 0
$$

where we used that $\log (x) \leq x-1$. This implies that $q(\tau)$ is decreasing. Along with the fact that $q(1)=0$, we have $q(\tau) \leq 0$ for all $\tau \geq 1$.

- Typos

1. In the last equation on the left column of page $7, \mathbb{P}\left(T_{1 j} \leq t-1\right)$ should be $\mathbb{P}\left(T_{k j} \leq t-1\right)$;
2. In the equation on the right column of page 9 regarding $\mathbb{P}\left(T_{1 n}^{n} \leq \tau\right), P$ should be $P_{1}$;
3. In the paragraph right before Remark 14 , one of $\mathbb{P}\left(T_{1 n}^{*} \leq \tau\right)$ should be $\mathbb{P}\left(T_{n 1}^{*} \leq \tau\right)$.
